HOMEWORK 2 - ANSWERS TO (MOST) PROBLEMS

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1. Section 1.6: Inverse functions and logarithms

1.6.3. No; For example, even though $2 \neq 6$, $f(2) = f(6) = 2$

1.6.5. Yes (by the horizontal line test)

1.6.16.

(a) $f^{-1}(3) = 0$
(b) $f(f^{-1}(5)) = 5$

1.6.18.

(a) By the horizontal line test
(b) Domain of f^{-1} = Range of $f = [-1, 3]$; Range of f^{-1} = Domain of $f = [-3, 3]$
(c) 0
(d) ≈ -1.8

1.6.26. $f^{-1}(x) = \ln\left(-\frac{x}{2x-1}\right) = \ln\left(\frac{x}{1-2x}\right)$

1.6.35.

(a) $\log_2(8) = 3$
(b) $\log_3\left(\frac{1}{9}\right) = -2$

1.6.36.

(a) $5^{-2} = \frac{1}{25}$
(b) 10

1.6.39. $\ln\left(\frac{(1+x^2)\sqrt{x}}{\sin(x)}\right)$

1.6.48.

(a) $x = \frac{\ln(7)-3}{2}$
(b) $x = \frac{5-e^{-3}}{2}$

1.6.58.

(a) $t = -a\ln\left(1 - \frac{Q}{Q_0}\right)$; Gives the time it takes to recharge the capacitor to a given capacity Q
(b) Plug in $Q = 0.9Q_0$ into the equation in (a), and you get $t = -2\ln(0.1) \approx 4.61$ seconds

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1.6.60.

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{3}$

1.6.64.

(a) $\frac{\sqrt{15}}{4}$
(b) $\frac{24}{25}$ (use the fact that $\sin(2x) = 2\sin(x)\cos(x)$)

1.6.65. If $\theta = \sin^{-1}(x)$, then $\sin(\theta) = x$, then draw a triangle with hypothenuse 1, and opposite side x, and then the adjacent side becomes $\sqrt{1-x^2}$, and so our answer becomes:

$$\cos(\sin^{-1}(x)) = \cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

See the handout "Proof of the derivative of arccos" for a similar problem; Or look at your notes taken in section!

1.6.66. $\tan(\sin^{-1} x) = \frac{\sin(\sin^{-1}(x))}{\cos(\sin^{-1}(x))} = \frac{x}{\sqrt{1-x^2}}$ by the result of number 65!

2. Section 2.2: The limit of a function

2.2.2. If x approaches 1 from the left, then $f(x)$ approaches 3; If x approaches 1 from the right, then $f(x)$ approaches 7. No, left-hand-limits and right-hand-limits must be equal!

2.2.6.

(a) 4
(b) 4
(c) 4
(d) Undefined
(e) 1
(f) -1
(g) Does not exist (left and right-side limits not equal)
(h) 1
(i) 2
(j) Undefined
(k) 3
(l) Does not exist (h does not approach one fixed value as x approaches 5 from the left)

2.2.28. $-\infty$ (numerator approaches $e^{-5} > 0$ while denominator approaches 0^-

2.2.29. $-\infty$ ($x^2 - 9$ approaches 0^+ and $\ln(0^+) = -\infty$

2.2.40. The mass blows up to ∞ ($\frac{v^2}{c^2}$ goes to 1^-, so the denominator of the fraction goes to 0^+, and so the whole fraction goes to ∞)

3. Section 2.3: Calculating limits using the limit laws

2.3.4. $\frac{9}{12} = \frac{3}{4}$

2.3.10.

(a) If you plug in $x = 2$, then the left hand side is not defined, but the right hand side is

(b) The above equation holds if $x \neq 2$, but the point of limits is that in this case you don't **care** about the value at 2! So in this case, the equality is correct!

2.3.13. Does not exist (left-hand-limit is $-\infty$ because the numerator tends to 4 and the denominator tends to 0^- while the right-hand-limit is ∞ because the numerator tends to 4 and the denominator tends to 0^+)

2.3.17. 8

2.3.18. 3 (use the fact that $x^3 - 1 = (x-1)(x^2 + x + 1)$)